Explicit models for bilateral fat-tailed distributions

and applications in finance with the package *FatTailsR*

patrice.kiener@inmodelia.com

R/Rmetrics Workshop - 27 June 2014

About InModelia

A small company located in Paris:

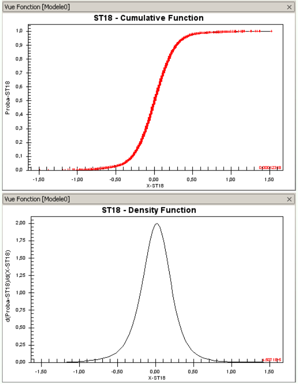
- Consulting services and training
- Modeling and software design

Since 2009:

- Neural networks
- Design of experiments for nonlinear models

Since 2013:

- Bilateral fat-tailed distributions
  (from a work done with ST-Microelectronics)



June 2014:

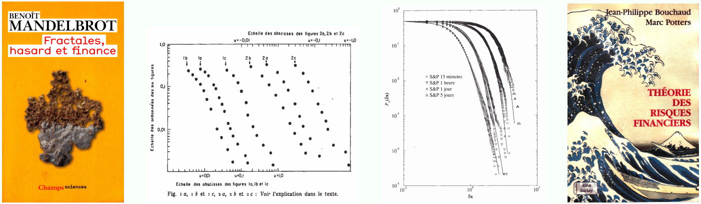
- Package **FatTailsR** to handle fat-tailed distributions in finance
  Models are symmetric (3 parameters) or asymmetric (4 parameters)
  Excellent results !!

Outline

1. Old models - New ideas
2. Symmetric models - Examples: Gold, Société Générale, Vivendi
3. Asymmetric models - Examples: S&P 500, Euro-Dollar, VIX
4. Package FatTailsR

Fat-tailed bilateral distributions

- Almost all datasets related to stock returns over long periods
- Market risk (Solvency II, Bâle III), portfolio management, derivatives, ...
- Mentionned by **Mandelbrot (1962)**, reviewed by **Bouchaud and Potters (1997)**, who use a combination of unilateral models for left and right tails



$$\log\{\mathrm{Fr}[L(t,T) > u)]\} \sim -\alpha \log u + \log C'(T).$$
$$\log\{\mathrm{Fr}[L(t,T) < -u)]\} \sim -\alpha \log u + \log C''(T).$$

$$L_\mu(x) \simeq \frac{\mu A_\pm^\mu}{|x|^{1+\mu}} \text{ pour } x \to \pm\infty,$$

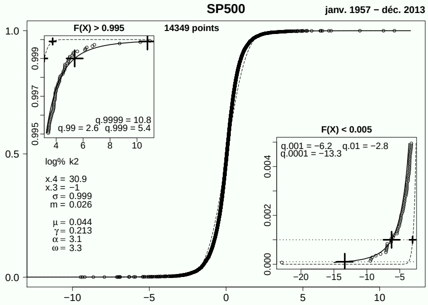
Figure : (a+b) Cotton price day-week-month - (c+d) Indice S&P 15min-day-week-month

- Centered and non-centered Student distributions
- Characteristic functions

$\Longrightarrow$ **We present a new, explicit, symmetric or asymmetric distribution**

Example: S&P 500

$\Longrightarrow$ **We present a new, explicit, symmetric or asymmetric distribution**



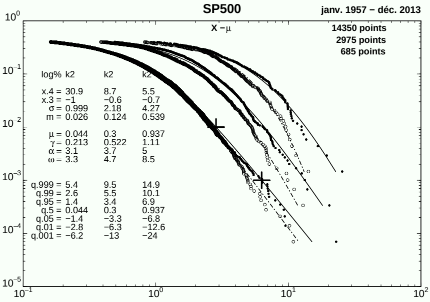


Figure : S&P 500 : (a) Cumulative function of the logreturns (b) Log-Log view day-week-month

New idea

Consider the logistic function which have thinner tails than the Laplace-Gauss function :

$$F(x) = \frac{1}{1 + e^{-x}}$$

It is remarkable that the combination of two asymmetric functions e^{-x} et $\frac{1}{1+\ldots}$ gives a perfectly symmetric model. This comes from the fundamental property of the exponential $e^{-x}e^{x} = 1$ which imposes $F(-x) = 1 - F(x)$ and for the density function $f(x) = F(x)F(-x)$

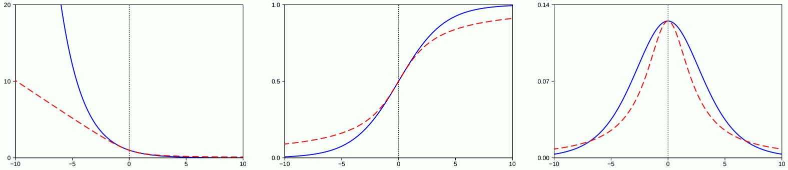


Figure : (a) Exp and hp (b) logis and logishp (c) dlogis and dlogishp

New idea

Consider the logistic function which have thinner tails than the Laplace-Gauss function :

$$F(x) = \frac{1}{1+e^{-x}}$$

It is remarkable that the combination of two asymmetric functions e^{-x} et $\frac{1}{1+\ldots}$ gives a perfectly symmetric model. This comes from the fundamental property of the exponential $e^{-x}e^{x} = 1$ which imposes $F(-x) = 1 - F(x)$ and for the density function $f(x) = F(x)F(-x)$

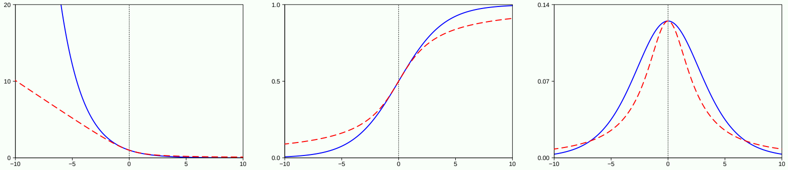


Figure : (a) Exp and hp (b) logis and logishp (c) dlogis and dlogishp

We use the other curves that verify the property $y(-x)y(x) = 1$:

- hyperbolas
- Power hyperbolas with parameter κ that allow the design of logistic type functions with tail convergence similar to $|x|^{-\kappa}$

Power hyperbolas

We call *power hyperbolas* the positive functions that verify:

Generic equation of power hyperbolas

$$\left(y^{1/\kappa} + \frac{X-\mu}{\gamma\kappa}\right)\ y^{1/\kappa} = 1 \qquad (1)$$

Solution to the equation are the curves:

$$y(X,\mu,\gamma,\kappa) = \left(-\frac{X-\mu}{2\gamma\kappa} + \sqrt{\left(\frac{X-\mu}{2\gamma\kappa}\right)^2 + 1}\right)^{\kappa} = e^{\kappa\ \log\left(-\frac{X-\mu}{2\gamma\kappa} + \sqrt{\left(\frac{X-\mu}{2\gamma\kappa}\right)^2 + 1}\right)}$$

or:

Power hyperbolas

$$y(X,\mu,\gamma,\kappa) = e^{-\kappa\ \mathrm{asinh}\left(\frac{X-\mu}{2\gamma\kappa}\right)} \quad \xrightarrow{\kappa\to+\infty} \quad e^{-\frac{X-\mu}{2\gamma}} \qquad (2)$$

When $\kappa = 1$, the curve is the simple hyperbola

Exphp, sinhp, coshp, tanhp

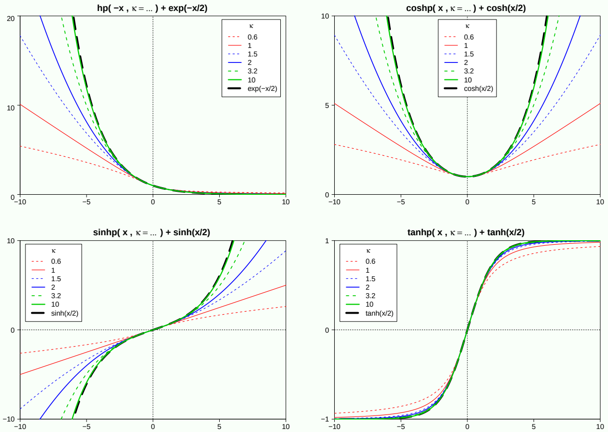


Figure : (a) Power hyperbola - exphp $_{\kappa}(-x) = e^{-\kappa \,\mathrm{asinh}(\frac{x}{2\kappa})} \xrightarrow{\kappa \to +\infty} e^{-\frac{x}{2}}$

(b) Power hyperbolic cosine - coshp $_{\kappa}(x) = \cosh\left[\kappa \,\mathrm{asinh}(\frac{x}{2\kappa})\right] \xrightarrow{\kappa \to +\infty} \cosh(\frac{x}{2})$

(c) Power hyperbolic sine - sinhp $_{\kappa}(x) = \sinh\left[\kappa \,\mathrm{asinh}(\frac{x}{2\kappa})\right] \xrightarrow{\kappa \to +\infty} \sinh(\frac{x}{2})$

(d) Power hyperbolic tangent - tanhp $_{\kappa}(x) = \tanh\left[\kappa \,\mathrm{asinh}(\frac{x}{2\kappa})\right] \xrightarrow{\kappa \to +\infty} \tanh(\frac{x}{2})$

Power hyperbolas, cumulative functions, densities K1

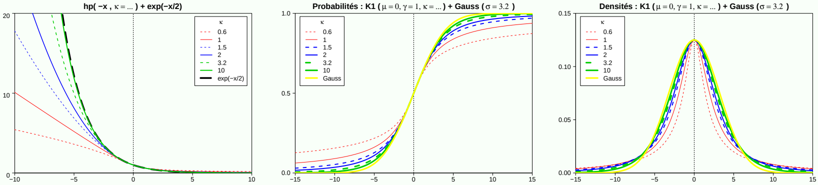


Figure : (a) Power hyperbolas - (b) Cumulative functions - (c) Densities

K1 model: cumulative functions and densities, $T = \text{asinh}\left(\frac{X-\mu}{2\gamma\kappa}\right)$:

$$F(X) = \frac{1}{1 + e^{-\kappa\ \text{asinh}\left(\frac{X-\mu}{2\gamma\kappa}\right)}} \qquad f(X) = \frac{1}{4\gamma \cosh(T)\ (1+\cosh(\kappa T))} \qquad f(0) = \frac{1}{8\gamma} \qquad (3)$$

F and f verify Karamata theorem related to slowly varying functions:

$$\lim_{x\to-\infty} \frac{x\,f(x)}{F(x)} = -\kappa \quad et \quad \lim_{x\to+\infty} \frac{x\,f(x)}{1-F(x)} = \kappa$$

Quantiles and densities (calculated from the probability):

$$X = \mu + 2\gamma\kappa \sinh\left(\frac{\text{logit}(p)}{\kappa}\right) \qquad f(X) = \frac{1}{2\gamma}\,\text{sech}\left(\frac{\text{logit}(p)}{\kappa}\right) p\,(1-p)$$

Logit and Logdensities

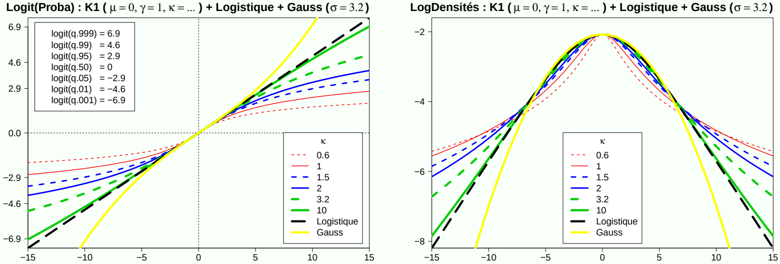


Figure : (a) QL-Plot = Quantiles + Logit of cumulative functions (b) Logdensities

$$\text{logit } F(X, \mu, \gamma, \kappa) = \kappa \text{ asinh}\left(\frac{X-\mu}{2\gamma\kappa}\right) \qquad \log f(X) = -\log(8\gamma) - \log(\cosh(T)) - 2\log\left(\cosh\left(\frac{\kappa T}{2}\right)\right)$$

Remarks on the QL-plot (a) with the logit of the cumulative function on y-axis:

- Logit-scale: $\text{logit}(0.001, 0.01, 0.05, 0.95, 0.99, 0.999) = (-6.9, -4.6, -2.9, 0, 2.9, 4.6, 6.0)$
- The logit of the logistic function $(\frac{x}{2})$ is the dashed bisecting line
- The Laplace-Gauss function (dashed lines+points) is concave and then convex
- The power hyperbola logistic functions are convex and then concave (=> subexponential functions)
- The curvatures of the power hyperbola logistic functions depend on parameter κ and are well separated

Comparison with Cauchy distribution ($\kappa = 1$)

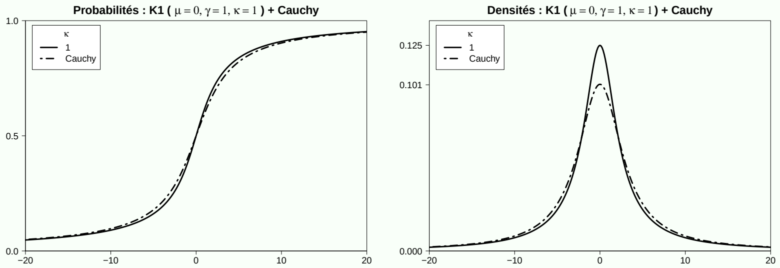


Figure : K1 model ($\kappa = 1$) and Cauchy functions: (a) Cumulative functions - (b) Densities

$$F(x) = \frac{1}{1-\frac{x}{2}+\sqrt{\left(\frac{x}{2}\right)^2+1}} \qquad x = \frac{2p-1}{p\,(1-p)} \qquad f(x) = \frac{1}{x^2+4+2\sqrt{x^2+4}} \qquad f(x) = \frac{1}{\left(\frac{1}{p}\right)^2+\left(\frac{1}{1-p}\right)^2}$$

$$G(x) = 0,5 + \frac{1}{\pi}\arctan\left(\frac{x}{\pi}\right) \qquad x = \pi\,\tan\left(\pi\,(p-0,5)\right) \qquad g(x) = \frac{1}{\pi^2+x^2} \qquad g(x) = \frac{\cos^2(\pi\,(p-0,5))}{\pi^2}$$

$$\lim_{X\to+\infty} F(X) = 1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^4} + o\left(\frac{1}{x^5}\right) \qquad \lim_{X\to+\infty} G(X) = 1 - \frac{1}{x} + \frac{\pi^2}{3x^3} - \frac{\pi^4}{5x^5} + o\left(\frac{1}{x^7}\right)$$

Comparison with Laplace-Gauss distribution (any κ, $\gamma = 0.313$)

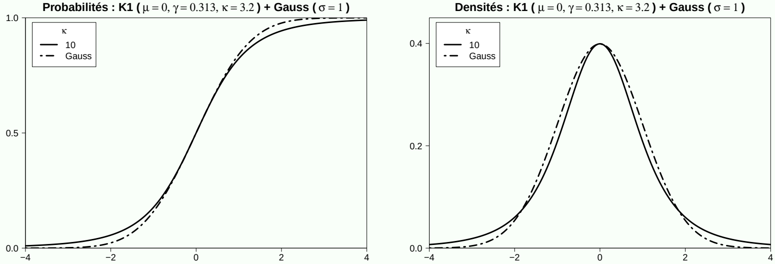


Figure : K1 model ($\gamma = 0,313$, $\kappa = 3.2$) and Laplace-Gauss distributions ($\sigma = 1$): (a) Cumulative functions - (b) Densities

K1 and Laplace-Gauss distributions can be compared when they have same density peaks.
The equality in $X = \mu$ gives $\hbar = \frac{\zeta_\sigma}{\zeta_\gamma} = \frac{8\,\gamma}{\sqrt{2\pi}\,\sigma} = 1$, or:

$$\gamma = \frac{\sqrt{2\pi}}{8}\,\sigma \approx 0,313\,\sigma \quad \text{et} \quad \sigma = \frac{8}{\sqrt{2\pi}}\,\gamma \approx 3,192\,\gamma$$

Comparison with centered Student distribution

Centered Student distribution with ν degrees of freedom:

$$f(x) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

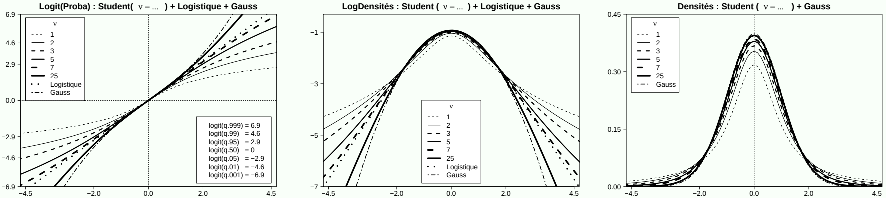


Figure : Student ($\nu = 1, 2, 3, 5, 7, 25$), Logistic and Laplace-Gauss distributions:
(a) Logit of cumulative functions - (b) Logdensities - (c) Densities

- ν can take integer values only:
  - $\nu \prec 1$ is impossible
  - $\nu = 1 \equiv$ Cauchy distribution. $\nu = 2 \approx \kappa = 2$. $\nu = 8 \approx$ logistic distribution
  - $\nu = +\infty \equiv$ Laplace-Gauss
- The density peaks depend on ν
- In pratice, one can use $\nu = (2, 3, 4, 5, 6, 7, 8)$ which cover the subexponential domain

Symmetric processes ($\kappa = 2$, $\kappa = 3.2$, $\kappa = 5$ et $\kappa = 10$) x ($\epsilon = 0$)

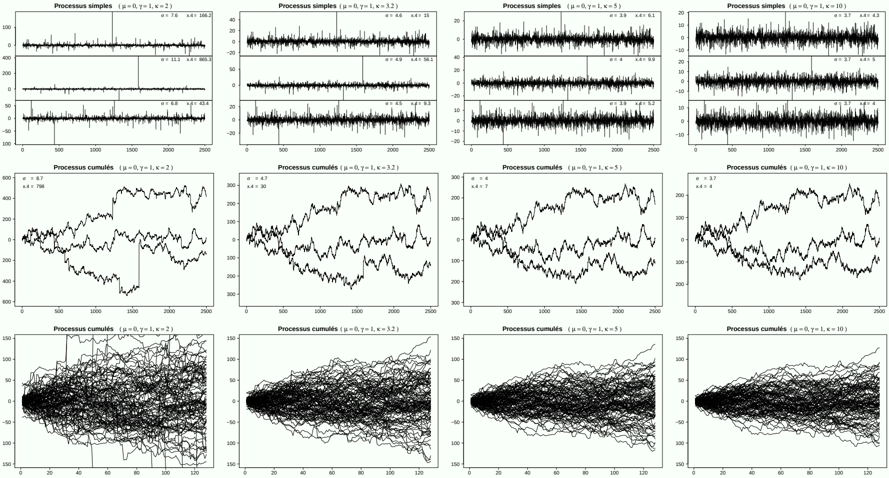


Figure : Processes and cumulated processes: (a) $\kappa = 2$ (b) $\kappa = 3.2$ (c) $\kappa = 5$ (c) $\kappa = 10$

Gold lingot: $\kappa = 4$, $\epsilon = 0$, multi-scale symmetry

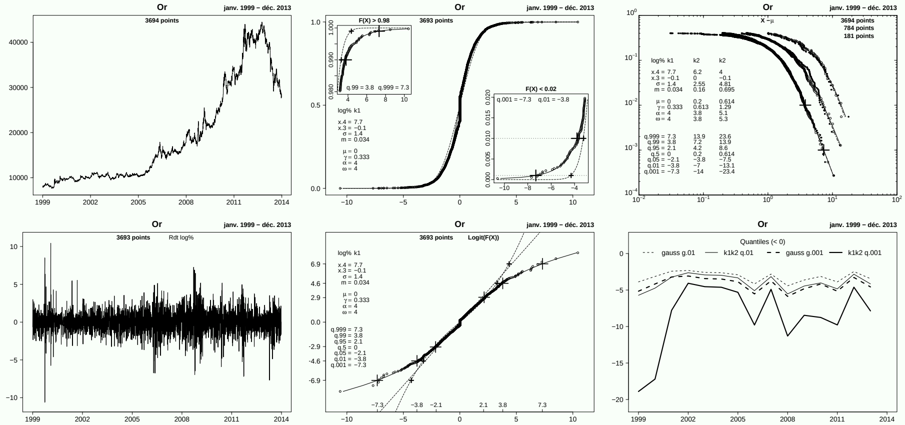


Figure : (a) Price of the gold lingot - (b) 100xlog-returns
(c) Cumulative function ($\kappa = \alpha = \omega$) - (d) Logit of the cumulative function
(e) Cumulative function in Log-Log scale with periods day, week, month
(f) Risks at 1% et 1‰ over a yearly period (250 days) described by Laplace-Gauss and K1 estimates

The annual risk profiles of Société Générale and Vivendi

The analysis of the returns can be conducted over fixed periods, for instance one year (about 250 days). The tail parameter κ describes the model curvature. q.001 is the risk at 1‰.

⇓

Société-Générale	x.4	σ	m	μ	γ	κ	q.001	q.01
1992	4.1	2.0	0.29	0.00	0.50	3.8	-11.5	-5.8
1993	3.3	1.4	0.09	0.13	0.38	12.0	-5.4	-3.4
1994	3.4	1.4	-0.12	-0.18	0.40	12.0	-5.9	-3.9
1995	4.9	1.7	0.03	-0.03	0.43	4.6	-8.5	-4.7
1996	3.9	1.3	-0.03	0.00	0.34	7.2	-5.5	-3.4
1997	4.3	1.9	0.15	0.21	0.51	5.4	-9.0	-5.1
1998	4.4	3.3	0.04	0.29	0.86	4.3	-17.1	-9.1
1999	6.0	2.5	0.20	0.23	0.60	3.3	-15.5	-7.2
2000	4.7	2.3	0.05	0.00	0.61	4.7	-11.8	-6.5
2001	4.6	2.3	-0.02	0.00	0.59	4.1	-12.6	-6.6
2002	4.6	3.4	-0.05	0.12	0.86	4.2	-18.1	-9.6
2003	4.3	2.1	0.09	0.00	0.57	5.3	-10.2	-5.9
2004	4.1	1.2	0.02	0.00	0.32	5.8	-5.6	-3.3
2005	3.9	1.0	0.13	0.11	0.28	6.5	-4.5	-2.6
2006	4.9	1.4	0.08	0.00	0.35	4.0	-7.7	-4.0
2007	4.7	1.8	-0.10	0.06	0.47	4.4	-9.4	-5.0
2008	5.2	4.5	-0.37	-0.48	1.08	3.4	-27.8	-13.7
2009	4.7	3.7	0.12	0.40	0.94	4.0	-20.0	-10.2
2010	11.9	3.1	-0.08	-0.10	0.67	2.8	-22.0	-9.4
2011	6.9	4.3	-0.33	-0.31	0.94	2.7	-32.6	-13.7
2012	3.7	3.3	0.20	0.20	0.91	12.0	-13.1	-8.4
2013	4.5	2.2	0.16	0.18	0.58	5.3	-10.3	-5.8
'92-'13	9.4	2.6	0.02	0.00	0.59	3.6	-14.2	-7.0

⇑

⇓

Vivendi	x.4	σ	m	μ	γ	κ	q.001	q.01
1992	3.2	1.8	0.00	-0.08	0.52	12.0	-7.6	-4.9
1993	2.9	1.4	0.12	0.07	0.40	12.0	-5.8	-3.7
1994	3.6	1.8	-0.14	-0.20	0.49	12.0	-7.4	-4.8
1995	3.2	1.8	-0.02	-0.19	0.51	12.0	-7.6	-5.0
1996	4.5	1.4	0.11	0.00	0.35	4.5	-7.0	-3.8
1997	3.3	1.6	0.11	0.13	0.46	12.0	-6.6	-4.2
1998	4.5	1.9	0.22	0.16	0.51	4.9	-9.4	-5.2
1999	3.0	1.9	0.08	0.00	0.52	12.0	-7.6	-4.9
2000	4.2	2.7	-0.10	-0.06	0.72	5.0	-13.5	-7.6
2001	6.0	2.6	-0.05	-0.14	0.63	3.3	-16.6	-7.9
2002	8.9	6.0	-0.54	-0.67	1.23	2.5	-49.0	-19.5
2003	5.1	2.8	0.09	0.00	0.71	4.0	-15.3	-8.0
2004	4.2	1.5	0.08	0.05	0.40	5.0	-7.4	-4.2
2005	3.7	1.1	0.05	0.08	0.30	10.7	-4.3	-2.7
2006	3.2	1.1	0.04	0.11	0.32	12.0	-4.5	-2.9
2007	3.7	1.1	0.02	0.00	0.30	7.9	-4.7	-3.0
2008	6.8	2.5	-0.12	-0.04	0.59	3.1	-16.7	-7.6
2009	3.6	1.7	-0.04	0.00	0.48	12.0	-7.0	-4.5
2010	9.9	1.7	-0.01	0.08	0.36	2.7	-12.4	-5.1
2011	4.0	1.9	-0.07	0.03	0.52	6.3	-8.7	-5.2
2012	6.9	1.8	0.00	-0.02	0.46	3.8	-10.5	-5.3
2013	3.7	1.5	0.05	0.06	0.40	8.3	-6.2	-3.8
'92-'13	25.5	2.2	-0.01	0.00	0.45	2.9	-14.2	-6.1

⇑

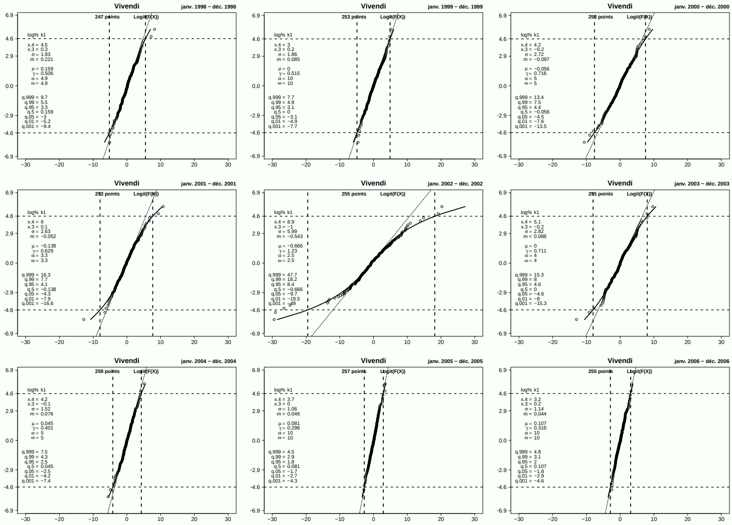


Figure : Logit of cumulative functions of Vivendi (2002: Jean-Marie Messier)
(a) 1998 - (b) 1999 - (c) 2000 - (d) 2001 - (e) 2002 - (f) 2003 - (g) 2004 - (h) 2005 - (i) 2006

The yearly return and risk profiles of Société Générale

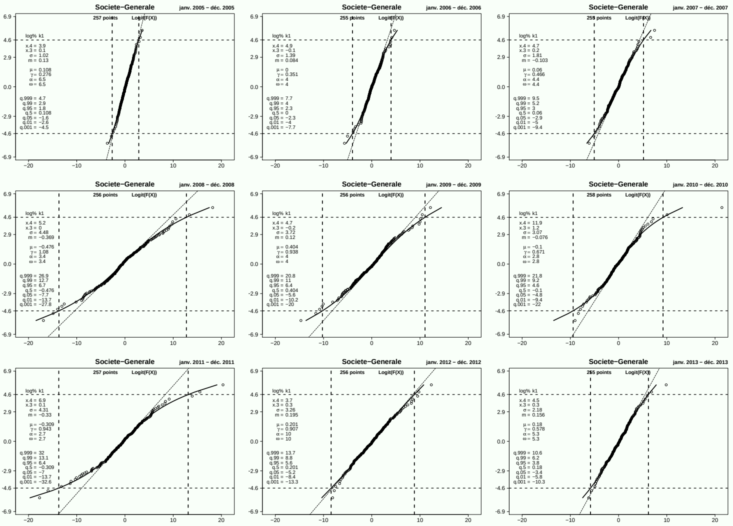


Figure : Logit of cumulative functions of Société Générale
(a) 2005 - (b) 2006 - (c) 2007 - (d) 2008 - (e) 2009 - (f) 2010 - (g) 2011 - (h) 2012 - (i) 2013

The yearly risk profiles

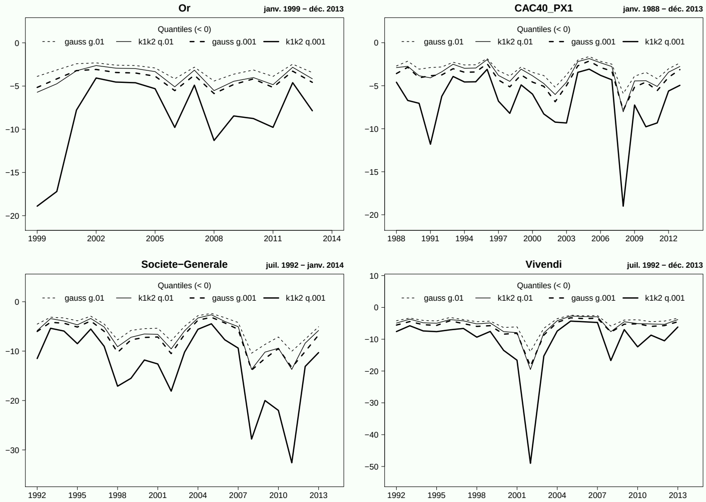


Figure : Quantiles: (a) Gold lingot - (b) CAC40 - (c) Société Générale - (d) Vivendi estimated with Laplace-Gauss (dashed lines) and K1 (full lines) models at levels 1% et 1‰ with datasets of about 250 points per year

Comparison of K1 distribution parameters

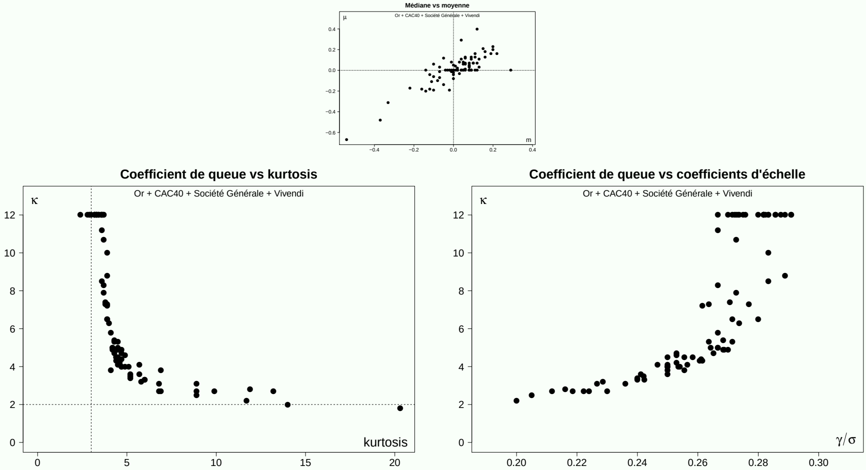


Figure : Comparison of various parameters obtained from *K1* distribution (one point = one year ≈ 250 days) : (a) Median and mean - (b) κ and kurtosis - (c) κ and ratio γ/σ

Asymmetric power functions - K2

We call *power functions* the positive functions that verify the equation:

Power functions

$$\left(y^{1/\alpha} + \frac{X-\mu}{\kappa\gamma}\right)\ y^{1/\omega} = 1 \qquad (4)$$

where κ is the harmonic mean of α and ω such that $\frac{1}{\kappa} = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\omega}\right)$

We call *distribution K2* the cumulative function F and the density function f defined by:

$$p = F(X) = \frac{1}{1+y} \quad et \quad f(X) = \frac{dF(X)}{dX} \qquad (5)$$

X has a simple form in y and p. since $y = \frac{1-p}{p}$ and $\log(y) = -\operatorname{logit}(p)$, it comes:

$$X = \mu + \gamma\kappa\left(-y^{1/\alpha} + y^{-1/\omega}\right) = \mu + \gamma\kappa\left(-\left(\frac{p}{1-p}\right)^{-1/\alpha} + \left(\frac{p}{1-p}\right)^{1/\omega}\right)$$

Quantile of K2 distribution

$$X(p;\mu,\gamma,\alpha,\omega) = \mu + \gamma\kappa\left(-e^{\frac{-\operatorname{logit}(p)}{\alpha}} + e^{\frac{\operatorname{logit}(p)}{\omega}}\right) \qquad (6)$$

α and ω are naturally highly correlated within model K2. Consider the therms ϵ and δ:

$$\frac{1}{\kappa} = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\omega}\right) \qquad \textit{et} \qquad \delta = \frac{\epsilon}{\kappa} = \frac{1}{2}\left(-\frac{1}{\alpha} + \frac{1}{\omega}\right)$$

It comes:

Conversion from α and ω to and from κ, δ and ϵ

$$\frac{1}{\alpha} = \frac{1}{\kappa} - \delta \qquad \frac{1}{\omega} = \frac{1}{\kappa} + \delta \qquad \epsilon = \frac{\alpha - \omega}{\alpha + \omega} \qquad \alpha = \frac{\kappa}{1-\epsilon} \qquad \omega = \frac{\kappa}{1+\epsilon} \tag{7}$$

$-1 < \epsilon < 1$ is a measure of the model **excentricity**. It can be expressed as a %

$\frac{-1}{\kappa} < \delta < \frac{1}{\kappa}$ is a measure of the model **distorsion**. It can be expressed in % or ‰

Rewrite the model K2:

Quantiles for models K3 and K4

$$\begin{cases} X(p; \mu, \gamma, \kappa, \delta) = \mu + 2\gamma\kappa \sinh\left(\frac{\text{logit}(p)}{\kappa}\right) \; e^{\delta \, \text{logit}(p)} \\ X(p; \mu, \gamma, \kappa, \epsilon) = \mu + 2\gamma\kappa \sinh\left(\frac{\text{logit}(p)}{\kappa}\right) \; e^{\frac{\epsilon}{\kappa} \, \text{logit}(p)} \end{cases} \tag{8}$$

Remark: $e^{\delta \, \text{logit}(p)} = \left(\frac{p}{1-p}\right)^{\delta}$

Asymmetric models (K2, K3, K4)

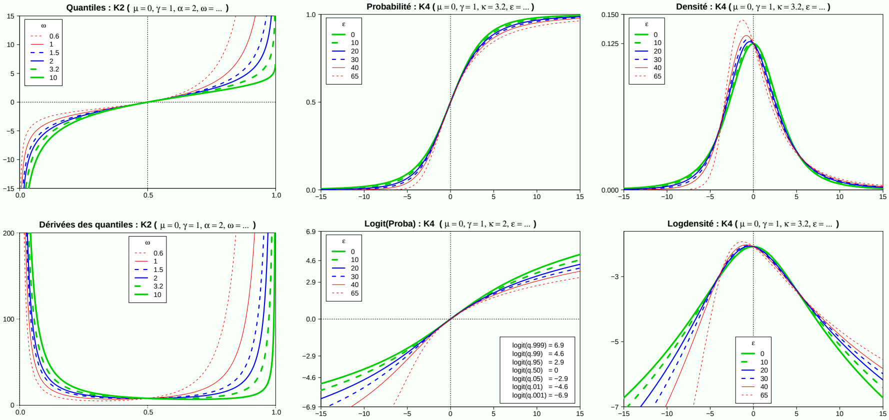


Figure : Models K2 and K3 : (a) Quantiles - (b) Cumulative functions (c) Densities (d) Quantiles derivates - (e) Logit of the cumulative functions - (f) Logdensities

Comparison with noncentral Student distribution

Noncentral Student distribution with ν degrees of freedom and noncentral parameter μ :

$$F_{v,\mu}(x) = \begin{cases} \frac{1}{2}\sum_{j=0}^{\infty} \frac{1}{j!}(-\mu\sqrt{2})^j e^{\frac{-\mu^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\Gamma(1/2)} I\left(\frac{v}{v+x^2}; \frac{v}{2}, \frac{j+1}{2}\right), & x \geq 0 \\ 1 - \frac{1}{2}\sum_{j=0}^{\infty} \frac{1}{j!}(-\mu\sqrt{2})^j e^{\frac{-\mu^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\Gamma(1/2)} I\left(\frac{v}{v+x^2}; \frac{v}{2}, \frac{j+1}{2}\right), & x < 0 \end{cases}$$

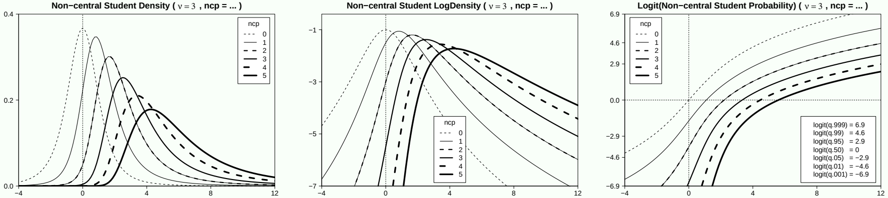


Figure : Noncentral Student distribution ($\nu = 3$, $ncp = 0, 1, 2, 4, 5$)
(a) Densities - (b) Logdensities - (c) Logit of the cumulative functions)

- The noncentral parameter μ can take any real value
- It **simultaneously** modifies the distribution tails AND the mode, the median and the mean

Asymmetric processes ($\kappa = 2,\ 3.2,\ 5,\ 10$)x($\delta = -0.04,\ -0.08,\ -0.12$)

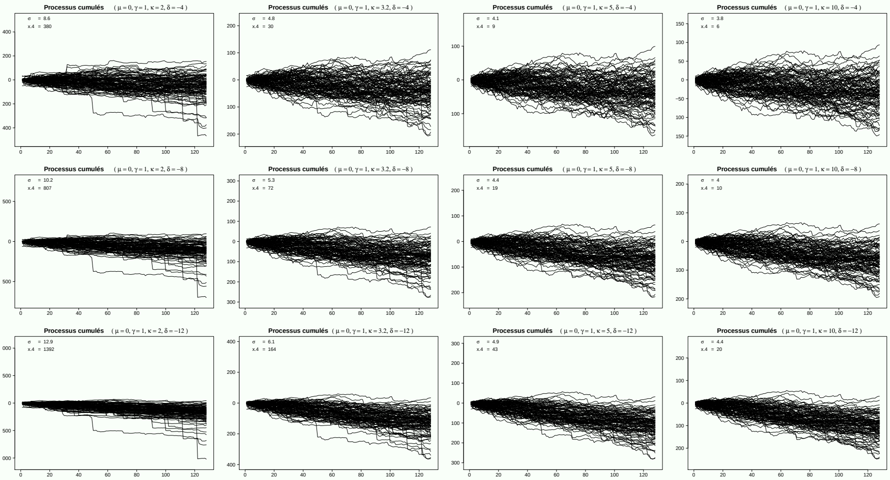


Figure : Cumulative processes: (a) $\kappa = 2$ (b) $\kappa = 3.2$ (c) $\kappa = 5$ (c) $\kappa = 10$

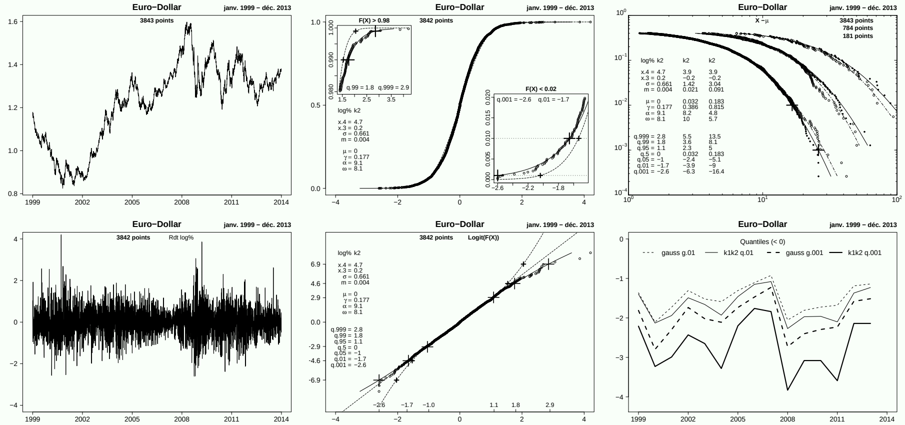


Figure : (a) Euro-Dollar price - (b) 100xlogreturns of Euro-Dollar
(c) Cumulative function ($\alpha > \kappa > \omega$) - (d) Logit of the cumulative function
(e) Cumulative function in Log-Log scale with periods day, week, month
(f) Risks at 1% et 1‰ over a yearly period (250 days) described by Laplace-Gauss and K2 estimates

VIX: $\alpha = 5$, $\omega = 4$, $\epsilon = 11\%$, strong multi-scale asymmetry

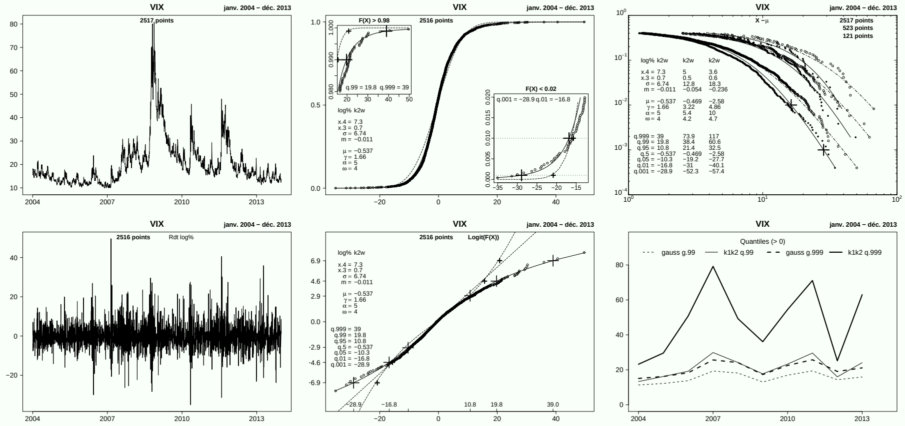


Figure : (a) VIX price - (b) 100xlogreturns of VIX
(c) Cumulative function ($\alpha > \kappa > \omega$) - (d) Logit of the cumulative function
(e) Cumulative function in Log-Log scale with periods day, week, month
(f) Risks at 1% et 1‰ over a yearly period (250 days) described by Laplace-Gauss and K2 estimates

Package FatTailsR

Miscellaneous

- ashp, kashp(x, k = 1), dkashp_dx, invlogit(x), logit(p)

Power hyperbolas, power hyperbolic functions and their inverses

- exphp(x, k = 1), coshp, sinhp, tanhp, sechp, cosechp, cotanhp
- loghp(x, k = 1), acoshp, asinhp, atanhp, asechp, acosechp, acotanhp

Logishp and Kiener1 symmetric functions, without or with parameter m and g

- d,p,q,r logishp(*xqpn*, k = 1)
- d,p,q,r kiener1(*xqpn*, m = 0, g = 1, k = 3.2)

Kiener2, Kiener3, Kiener4 asymmetric functions

- q,r kiener2(*pn*, m = 0, g = 1, a = 3.2, w = 3.2)
- q,r kiener3(*pn*, m = 0, g = 1, k = 3.2, d = 0)
- q,r kiener4(*pn*, m = 0, g = 1, k = 3.2, e = 0)

Parameter estimation

- laplacegaussnorm(X)
- regkienerLX(X, model = "k4", dgts = c(3, 3, 1, 1, 1, 3, 2, 4, 4, 2, 2),
  maxk = 10, mink = 0.7, app = 0)

$\Longrightarrow$ **Available on CRAN at:**

`http://cran.r-project.org/web/packages/FatTailsR/index.html`

Package FatTailsR: *regkienerLX* and SMI indice

```
> library(FatTailsR)
> price2returns <- function(x) { 100*diff(log(x)) }
> j   <- 2   # 1=DAX, 2=SMI, 3=CAC, 4=FTSE
> X   <- price2returns(EuStockMarkets[,2])
> reg <- regkienerLX( X, model = "k2" )
> attributes(reg)
$names
 [1] "dfrXP"    "dfrXL"    "dfrXR"    "dfrEP"    "dfrEL"    "dfrED"    "regk0"    "coefk0"   "vcovk0"
[10] "vcovk0m"  "mcork0"   "coefk"    "coefk1"   "coefk2"   "coefk3"   "coefk4"   "quantk"   "coefr"
[19] "coefr1"   "coefr2"   "coefr3"   "coefr4"   "quantr"   "dfrQkPk"  "dfrQkLk"

$class
[1] "clregk"

> reg$coefr2
    m     g     a     w
0.089 0.223 3.800 4.400

> reg$coefr
     m      g      a      k      w      d      e
 0.089  0.223  3.800  4.100  4.400 -0.018 -0.070

> reg$quantr
q.0001 q.0005  q.001  q.005   q.01   q.05   q.50   q.95   q.99  q.995  q.999 q.9995 q.9999
-10.00  -6.44  -5.30  -3.29  -2.63  -1.42   0.09   1.44   2.39   2.88   4.29   5.06   7.33
```

Conclusion

We have presented:

- Power hyperbolas, power hyperbolic functions and fat-tailed distributions which use the **median** as pivotal value
  - A symmetric model whose all representations have explicit forms (pdf, cdf, quantiles, ...)
  - A few asymmetric models whose quantile functions have explicit forms
- Very accurate models for distributions of returns. Accurate estimates of risks
- A new R package: **FatTailsR**

Some open questions:

- **Characteristic functions**, moments, Bayesian prior and posterior
- Multivariate models, copules. But which **scalar product** and which **correlation** ?

  Correlation of central parameter γ, tail parameter κ or quantile value $q_{1\%}$?
- **Stochastic processes**. Here, $\mu\gamma\kappa\epsilon$ are not $m\sigma$!!
  What name for these new processes: $\mu\gamma\kappa\epsilon$-Garch or maybe Garck ?

$$\frac{dS_t}{S_t} = \mu_m dt + \gamma dW_t \quad \text{with} \quad dW_t \approx\approx 2\kappa \sinh\left(\frac{\text{logit}(p_t)}{\kappa}\right) e^{\frac{\epsilon}{\kappa}\text{logit}(p_t)}$$

- Could the nonlinear variation of parameters (γ/σ , κ) presented at slide 20 explain the **volatility smile** of derivatives products?

Potential applications:

- Market risk (Bâle III, bcbs240, Jan-Fev. 2013), portfolio management, derivatives, ...
- Add new features to the package

References

1. B. Mandelbrot, Sur certains prix spéculatifs : faits empiriques et modèle basé sur les processus stables additifs non gaussiens de Paul Lévy, C. R. Acad. Sci. Paris, vol. 254, (1962) 3968-3970 (reprinted in Fractales, hasard et finance, Flammarion, 2009).
2. J-P. Bouchaud, M. Potters, Théorie des risques financiers, CEA Aléa Saclay, 1997.
3. L. De Haan, On regular variation and its application to the weak convergence of sample extremes, Mathematical Centre Tracts vol. 32, Matematisch Centrum Amsterdam, 1970.
4. The R Project for Statistical Computing, http://www.r-project.org
5. Rmetrics, http://www.rmetrics.org
6. 8th R/Rmetrics Workshop and Summer School, https://sites.google.com/site/rmetricsparis2014/

Package **FatTailsR**

- Version 1.0-3 (14 July 2014) available on CRAN at: http://cran.r-project.org/web/packages/FatTailsR/index.html

To cite and download this document:

- P. Kiener, Explicit models for bilateral fat-tailed distributions and applications in finance with the package FatTailsR, 8th R/Rmetrics Workshop and Summer School, Paris, 27 June 2014. http://www.inmodelia.com/fattailsr-en.html

Thank you for your attention !

patrice.kiener@inmodelia.com

Tél. : +33.9.53.45.07.38